Nexus International Journal of Science & Technology

Vol. 15, №. 1, 2026 | pp. 116–124

$\mathcal{P}_{mcv}$-MEASURE IN THE CLASS OF m-CONVEX FUNCTIONB

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Article type	**Received**	**Accepted**	**Published**
Original Research	05 February 2026	12 February 2026	01 March 2026

ABSTRACT. This paper investigates key objects within the theory of m-convex $(m-cv)$ functions, specifically focusing on mcv-polar sets, $\mathcal{P}_{mcv}$-measures, and capacitive values. Drawing analogies to harmonic measures in classical potential theory, the author establishes that $\mathcal{P}_{mcv}$-measures exhibit extreme properties within the class of m-convex functions. The work characterizes the properties of mcv-polar sets, outlining their relation to Lebesgue and Hausdorff measures. Based on the Trudinger-Wang theorem, the existence criteria for these sets are examined under the conditions $m<\frac{n}{2}+1$ and $m\geq\frac{n}{2}+1$. Furthermore, the concept of $\mathcal{P}_{mcv}$-capacity for a condenser is introduced, and its fundamental properties – such as monotonicity, countable subadditivity.

Keywords. *m-convex function, mcv-polar set, $\mathcal{P}_{mcv}$-measure, fundamental function, $\mathcal{P}_{mcv}$-capacity.*

DOI: *https://zenodo.org/records/20510189* **How to cite:** *Ismoilov M.B. (2026). $\mathcal{P}_{mcv}$-measure in the class of m-convex functionb. Nexus International Journal of Science and Technology, 15(1), 116–124.*

1. INTRODUCTION

Let $u(x)\in C^2(D)$ be a twice smooth function in the domain $D\subset\mathbb{R}^n$. Then the matrix $\left(\frac{\partial^2 u}{\partial x_j \partial x_k}\right)$ is symmetric, $\frac{\partial^2 u}{\partial x_j \partial x_k}=\frac{\partial^2 u}{\partial x_k \partial x_j}$. Therefore, after a suitable orthonormal transformation, it can be transformed into a diagonal form

$$\left(\frac{\partial^2 u}{\partial x_j \partial x_k}\right)\rightarrow\begin{pmatrix}\lambda_1 & 0 & \dots & 0\\ 0 & \lambda_2 & \dots & 0\\ \dots & \dots & \dots & \dots\\ 0 & 0 & \dots & \lambda_n\end{pmatrix},$$

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DOI: `10.5281/zenodo.20510189`

where $\lambda_j = \lambda_j(x) \in \mathbb{R}$ are the eigenvalues of the matrix $\left(\frac{\partial^2 u}{\partial x_j \partial x_k}\right)$. Let

$$H^k(u) = H^k(\lambda) = \sum_{1 \le j_1 < \ldots < j_k \le n} \lambda_{j_1} \ldots \lambda_{j_k}$$

be the Hessian of dimension k of the vector $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)$.

Definition 1. *A twice smooth function* $u(x) \in C^2(D)$ is called m-convex in $D \subset \mathbb{R}^n$, $u \in m - cv(D)$, if its eigenvalue vector $\lambda = (\lambda_1(x), \lambda_2(x), \ldots, \lambda_n(x))$ satisfies the conditions

$$m - cv \cap C^2(D) = \left\{H^k(u) = H^k(\lambda(x)) \ge 0,\ \forall x \in D,\ k = 1, 2, \ldots, n - m + 1\right\}.$$

When $m = n$ the class $n - cv$ coincides with the class of subharmonic functions $sh = \{\lambda_1 + \lambda_2 + \ldots + \lambda_n \ge 0\}$, when $m = 1$ it coincides with the class of convex functions $cv = \{\lambda_1 \ge 0, \lambda_2 \ge 0, \ldots, \lambda_n \ge 0\}$, moreover $cv = 1 - cv \subset 2 - cv \subset \ldots \subset n - cv = sh$. The theory of subharmonic functions is a developed and important part of theory functions and mathematical physics. The theory of convex functions is well studied and reflected in the works of A. Aleksandrov, I. Bakelman, A.Pozdnyak and others (see [2-5]). When $m > 1$ this class was studied in the series of works by N. Ivochkina, N.Trudinger, X.Wang et al. [10, 14–16] (see also [7]).

If we want to construct a good theory of $m - cv$ functions, then the class of functions $C^2(D)$ is not enough. For example, if we want to solve the equation

$$\begin{aligned} &H^{n-m+1}(u) = f(u, x) \\ &u|_{\partial D} = \varphi \end{aligned},$$

or want to work with extreme $m - cv$ functions, such as maximal $m - cv$ functions, we need to extend the definition of $m - cv$ functions to a wider class of upper semi-continuous functions. In the work of N.Trudinger, X.Wang [16] $m - cv$ functions are introduced in the class of upper semi-continuous functions $u(x)$ in the domain $D \subset \mathbb{R}^n$, using the so-called “viscous” definition, that is $H^k(q) \ge 0,\ k = 1, 2, \ldots, n - m + 1,$ for any quadratic polynomial $q(x)$, such that the difference $u(x) - q(x)$ has only a finite number of local maximum in the domain D. In addition, in this work $H^{n-m+1}(u)$ (maximum degree operator) is defined as a Borel measure and with the help of this operator the capacity of condenser $C(E,D)$ was introduced, a number of potential properties of this capacity was proved.

To expand the domain of definition of $m-cv$ functions from $C^2(D)$ to a wider class of semi-continuous functions, we have proposed a completely new approach, the connection of $m-cv$ functions with m-subharmonic (sh_m) functions in complex space $\mathbb{C}^n$. The theory of sh_m-functions is well developed and is currently subject of study by many mathematicians (Z.B locki [6], S. Dinew and S.Kolodziej [8, 9], S. Y. Li [11], H.C.Lu [12,13] and etc). Quite a complete overview of this theory is available in the survey article by A. Sadullaev and B. Abdullaev [1] in proceedings of Mathematical Institute of the RAS.

In this set, $\mathcal{P}_{mcv}$-measures $\omega^*(x,E,D)$ and mcv-capacitive quantities $\mathcal{P}_{mcv}(E,D) = -\int\limits_D \omega^*(x,E,D)dV$ and the capacity of a condenser $C(E,D)$ in the class of m-convex functions. Analogous to harmonic measures in classical potential theory $\mathcal{P}_{mcv}$-measures possess the extremality property in the class of m-convex functions. They are involved in many questions of function theory, in uniform estimates of $m-$convex functions, in studies of solutions to the Dirichlet problem.

2. mcv-POLAR SETS.

mcv-polar sets are introduced similarly to polar or pluripolar sets in pluripotential theory [17].

Definition 1. *A set $E \subset D \subset \mathbb{R}^n$ is called a $mcv-polar$ set in D if there exists a function $u(x) \in m-cv(D)$, $u(x) \not\equiv -\infty$, such that $u|_E = -\infty$.*

From the embedding $cv = 1-cv \subset 2-cv \subset \ldots \subset m-cv \subset \ldots \subset n-cv = sh$ that, $m-cv(D) \subset L^1_{loc}(D)$ it follows that every mcv-polar set is polar in the sense of classical potential theory. In particular, for a mcv-polar set, E the Hausdorff measure $\mathrm{H}_{n-2+\varepsilon}(E) = 0$ $\forall \varepsilon > 0$, and therefore, the Lebesgue measure of the mcv-polar set E is also equal to zero. Moreover, by the Trudinger-Wong theorem [15] it follows that any m-convex function $u(x) \in m-cv$ for $m < \frac{n}{2}+1$ is Holder continuous with exponent $\alpha = 2 - \frac{n}{n-m+1}$, $u(x) \in Lip_\alpha(D)$. Therefore, for , $m < \frac{n}{2}+1$ a nonempty mcv-polar set does not exist; even a singleton set $\{x^0\} \subset D \subset \mathbb{R}^n$ is not mcv-polar . However, for, $m \geq \frac{n}{2}+1$ there is a rich set of mcv-polar sets.

Example 1. (fundamental $m-cv$ function).

$$\chi_m(x,0)=\begin{cases} |x|^{2-\frac{n}{n-m+1}} \quad если \;\; m<\left[\frac{n}{2}\right]+1 \\ \ln|x| \quad\;\; если \quad\; m=\left[\frac{n}{2}\right]+1 \\ -|x|^{2-\frac{n}{n-m+1}} \quad если \;\; m>\left[\frac{n}{2}\right]+1 \end{cases} \tag{1}$$

Thus, when $m<\frac{n}{2}+1$ the fundamental function is bounded and Lipschitz continuous, and when $m\geq\frac{n}{2}+1$ it is equal to $-\infty$ at the point $x=0$. Note that for $m=n$ i.e., for the subharmonic case, it coincides with the fundamental solution $-\frac{1}{|x|^{n-2}}$ Laplace operator Δ.

For further study of mcv-polar sets we will need special regular domains.

Definition 2. *A domain* $D\subset\mathbb{R}^n$ *is called* mcv *convex if there exists* $\rho(x)\in m-cv(D)$ *such that* $\lim\limits_{x\to\partial D}\rho(x)=+\infty$ *and is called strongly* mcv*-convex or* mcv*-regular if there exists* $\rho(x)\in m-cv(D)$: $\rho(x)<0$ *and* $\lim\limits_{x\to\partial D}\rho(x)=0$.

Definition 3. *A domain* $D\subset\mathbb{R}^n$ *is called strongly* mcv*-convex if* $D=\{\rho(x)<0\}$, *where is* $\rho(x)-$ *a continuous* $m-cv$ *function in some neighborhood of* $G\supset\overline{D}$. *In the case where here is a* $\rho(x)-$*strictly convex* $m-cv$ *function in ,* G, *then the domain* D *is called strongly* mcv*-convex .*

Recall that a function $\rho(x)$ is called a strictly $m-cv$ functional function in G, if for some $\varepsilon>0$ the difference $\rho(x)-\varepsilon|x|^2$ is $m-cv$ a function. The following two theorems are analogs of the corresponding theorems in classical and complex potential theory.

Theorem 1. *The countable union* of mcv*-polar sets* is mcv*-polar, i.e. if* $E_j\subset D$ *are* mcv-polar , *then* $E=\bigcup\limits_{j=1}^{\infty}E_j$ *is also* mcv-polar .

Theorem ***2.*** *Let* $D\subset\mathbb{R}^n$ $mcv-$ *a convex domain and a subset* $E\subset D$ *such that for any compact subdomain* $G\Subset D$ *the set* $E\bigcap G$ *is* $mcv-$*polar in* G. *Then* E *is* $mcv-$*polar in.* D. *Furthermore, if* D *is strongly* $mcv-$ *convex domain, then there exists a function* $u(x)\in m-cv(D)$, $\;u\big|_D<0$, $u\not\equiv-\infty$, *But* $u\big|_E\equiv-\infty$.

3. $\mathcal{P}_{mcv}$ -MEASURE.

$\mathcal{P}_{mcv}$-measure is a real analogue of $\mathcal{P}$-measures in pluripotential theory [1].

Let be $E\subset D$ *an arbitrary set in the domain* $D\subset\mathbb{R}^n$, $1\leq m\leq n$. *For simplicity, we assume that* D *is a strongly* mcv*-convex domain. Consider the class of functions* $\mathcal{U}(E,D)=\left\{u\in m-cv(D):u\big|_D\leq0,\ u\big|_E\leq-1\right\}$ *and lets put* $\omega(y,E,D)=\sup\{u(y):\ u\in\mathcal{U}(E,D)\}$.

Definition 4. *Regularization* $\omega^*(x,E,D)=\overline{\lim_{y\to x}}\,\omega(y,E,D)$ *is called the* $\mathcal{P}_{mcv}$*-measure (*$m-cv$ *measure) of the set E with respect to the domain* D*.*

Let us present some simple properties of the $\mathcal{P}_{mcv}$-measure.

1. (monotony) *If* $E_1\subset E_2$, *then* $\omega^*(x,E_1,D)\ge\omega^*(x,E_2,D)$; *if* $E\subset D_1\subset D_2$, *then* $\omega^*(x,E,D_1)\ge\omega^*(x,E,D_2)$. *Follows from class embeddings* $\mathcal{U}(E_1,D)\supset\mathcal{U}(E_2,D)$, $\mathcal{U}(E,D_1)\supset\mathcal{U}(E,D_2)$;

2. $\omega^*(x,U,D)\in\mathcal{U}(U,D)$ *for open sets* $U\subset D$ *and therefore* $\omega^*(x,U,D)\equiv\omega(x,U,D)$;

3. *If is* $U\subset D$ *an open set and* $U=\bigcup_{j=1}^{\infty}K_j$, *where* $K_j\subset K_{j+1}^0$ *are compact, then* $\omega^*(x,K_j,D)\downarrow\omega^*(x,U,D)$. *If is* $E\subset D$ *an arbitrary set, then there exists a decreasing sequence of open sets* $U_j\supset E$, $U_j\supset U_{j+1}$ $(j=1,2,\ldots)$ *such that* $\left(\lim_{j\to\infty}\omega\left(x,U_j,D\right)\right)^*=\omega^*\left(x,E,D\right)$.

The following theorem relates $\omega^*(x,E,D)\equiv0$ to mcv-polar sets.

Theorem 3. $\mathcal{P}_{mcv}$ *-measure* $\omega^*(x,E,D)$ *is either nowhere equal to zero or is identically equal to zero.* $\omega^*(x,E,D)\equiv0$. *Moreover,* $\omega^*(x,E,D)\equiv0$ *if and only if* E mcv*-polar in* D*.*

Indeed, if m-convex function $\omega^*(x,E,D)$ vanishes at some interior point, $x^0\in D$, then by the maximum principle $\omega^*(x,E,D)\equiv0$.

Furthermore, if $\omega^*(x,E,D)\equiv0$, then it is easy to find a point $x^0\in D$ such that $\omega(x^0,E,D)=0$. According to the definition, $0=\omega(x^0,E,D)=\sup\{u(x^0):\ u\in\mathcal{U}(E,D)\}$ there is a sequence $u_j\in\mathcal{U}(E,D):\ u_j\left(x^0\right)>-\frac{1}{2^j}$. Let's put $w\left(x\right)=\sum_{j=1}^{\infty}u_j\left(x\right)$. Since, all $u_j<0$, that $w\left(x\right)\in m-cv\left(D\right)$. Since $u_j\left(x^0\right)>-\frac{1}{2^j}$, that $w\left(x^0\right)>\sum_{j=1}^{\infty}\frac{1}{2^j}=-1$, i.e. $w\left(x\right)\not\equiv-\infty$. Since that $u_j|_E=-1$, From $w\left(x\right)\equiv-\infty$, $\forall x\in E$. This it follows that E is a mcv-polar set in D.

Conversely, if E is a mcv-polar set, D, then according to Theorem 2 there exists a function $u(x)\in m-cv(D)$, $u\big|_D<0$, $u\not\equiv-\infty$, but $u\big|_E\equiv-\infty$. Then the functions $u_j\left(x\right)=\frac{1}{j}u\left(x\right)\in\mathcal{U}(E,D)$, $j=1,2,\ldots$ *and* $\omega(y,E,D)=\sup\{u(y):\ u\in\mathcal{U}(E,D)\}=0$ for all $y\in D:\ u\left(y\right)\neq-\infty$. However, the set $\left\{y\in D:\ \ u\left(y\right)=-\infty\right\}$ has Lebesgue measure zero, i.e., it is nowhere dense. Therefore, $\omega^*(x,E,D)=\overline{\lim_{y\to x}}\,\omega\left(y,E,D\right)\equiv 0$. *The theorem is proved.*

4. (two constants theorem). *If in the area* $D\subset\mathbb{R}^n$ *function* $u(x)\in m-cv$ *and* $u\big|_D\le R,\ u\big|_E\le r,\ (E\subset D,\ r<R)$, *then for all* $x\in D$ *the inequality holds* $u(x)\le R(1+\omega^*(x,E,D))-r\omega^*(x,E,D)$.

5. *If a set* E *compactly lies in a strongly* mcv *convex domain,* $D=\left\{\rho\left(x\right)<0\right\}$, $E\Subset D$, *then the* $\mathcal{P}_{mcv}$ *-measure* $\omega^*(x,E,D)$ $m-cv$ *continues in .*

Definition 5. *A point is called regular point of a compact set if . A compact set is called -regular compact set if each of its points is -regular.*

The following theorem is very important in the study of functions.

Theorem 4. *If is a regular compact set, then -measure* is *and is a continuous function in .*

4. E CAPACITY QUANTITY IN THE CLASS OF m-CONVEX FUNCTIONS.

To have a metric characteristic of sets $E\subset D$ associated with capacity, we need to introduce the corresponding capacity quantity

Let the set $E\subset D$ and $\omega^*(x,E,D)$ its $\mathcal{P}_{mcv}$-measure be the magnitude.

$$\mathcal{P}_{mcv}(E,D)=-\int_D \omega^*(x,E,D)dV$$

is called the $\mathcal{P}_{mcv}$-capacity of the set E with respect to D.

Thus, the $\mathcal{P}_{mcv}$-capacitance expresses the capacitive value of the pair. $\left(E,D\right)$. Such a pair is usually called a capacitor in $\mathbb{R}^n$.

$\mathcal{P}_{mcv}$ - the capacity has the following properties:

1. $\mathcal{P}_{mcv}(E,D)=-\int_D \omega(x,E,D)dV$;

It follows from the fact that the set $\left\{x\in D:\omega(x,E,D)<\omega^*(x,E,D)\right\}$ has zero Lebesgue measure.

2. $\mathcal{P}_{mcv}(E,D)\ge 0$ and $\mathcal{P}_{mcv}(E,D)=0$ if and only if E is a mcv polar set in D.

The following theorem is key in the characterization of $\mathcal{P}_{mcv}$-capacity.

Theorem 5. *The quantity* $\mathcal{P}_{mcv}(E,D)$ *is an increasing and countably subadditive function of the set:* $\mathcal{P}_{mcv}(E_1,D)\le\mathcal{P}_{mcv}\left(E_2,D\right)$ *for* $E_1\subset E_2$ *and*

$$\mathcal{P}_{mcv}\left(\bigcup_{j=1}^{\infty}E_j,D\right)\le\sum_{j=1}^{\infty}\mathcal{P}_{mcv}\left(E_j,D\right). \tag{4}$$

Moreover, $\mathcal{P}_{mcv}(E,D)$ *it is continuous on the right, i.e. for any set* $E\subset D$ *and any* $\varepsilon>0$ *there exists an open set* $U\supset E$ *such that* $\mathcal{P}_{mcv}\left(U,D\right)-\mathcal{P}_{mcv}\left(E,D\right)<\varepsilon$.

Proof. Monotone $\mathcal{P}_{mcv}(E,D)$ follows obviously from the monotonicity property of the $\mathcal{P}_{mcv}$ -measure. The proof of (4) follows from a similar inequality $-\omega\left(x,\bigcup_{j=1}^{\infty}E_j,D\right)\le-\sum_{j=1}^{\infty}\omega\left(x,E_j,D\right)$ for $\mathcal{P}_{mcv}$ -measures: for any set, $u_j\left(x\right)\in\mathcal{U}\left(E_j,D\right)$ the sum $\sum_{j=1}^{\infty}u_j\left(x\right)$ is $m-cv$ function in the wide sense (it can also be equal to $-\infty$). In addition $\sum_{j=1}^{\infty}u_j\left(x\right)\in\mathcal{U}\left(\bigcup_{j=1}^{\infty}E_j,D\right)$, and, therefore $\sum_{j=1}^{\infty}u_j\left(x\right)\le\omega\left(x,\bigcup_{j=1}^{\infty}E_j,D\right)$. On the other hand,

$$\sup\left\{\sum_{j=1}^{\infty}u_j\left(x\right):\ u_j\left(x\right)\in\mathcal{U}\left(E_j,D\right)\right\}=$$

$$=\sum_{j=1}^{\infty}\sup\left\{u_j\left(x\right):\ u_j\left(x\right)\in\mathcal{U}\left(E_j,D\right)\right\}=\sum_{j=1}^{\infty}\omega\left(x,E_j,D\right)$$

and

$$\sum_{j=1}^{\infty}\omega\left(x,E_j,D\right)\le\omega\left(x,\bigcup_{j=1}^{\infty}E_j,D\right).$$

Integrating this inequality and using Levi's theorem on the integration of a monotone sequence, we obtain

$$-\int\omega\left(x,\bigcup_{j=1}^{\infty}E_j,D\right)dV\le-\sum_{j=1}^{\infty}\int\omega\left(x,E_j,D\right)dV.$$

It remains to show the continuity of the set function on the right $\mathcal{P}_{mcv}(E,D)$. We fix a set $E\subset D$ and, according to the property 3) of the $\mathcal{P}_{mcv}$ -measure, construct a sequence of open sets $U_j\supset E$, $U_j\supset U_{j+1}$: $\left[\lim_{j\to\infty}\omega\left(x,U_j,D\right)\right]^*\equiv\omega*\left(x,E,D\right)$. Since, $\omega\left(x,U_j,D\right)$ is increasing, then again by Levy's theorem

$$\lim_{j\to\infty}\mathcal{P}_{mcv}\left(U_j,D\right)=-\lim_{j\to\infty}\int\omega\left(x,U_j,D\right)dV=-\int\lim_{j\to\infty}\omega\left(x,U_j,D\right)=$$

$$=-\int\left[\lim_{j\to\infty}\omega\left(x,U_j,D\right)\right]^*dV=\mathcal{P}_{mcv}\left(E,D\right).$$

Hence, for any $\varepsilon>0$, there exists j_0 such that for $j\ge j_0$, the inequality holds $\mathcal{P}_{mcv}\left(U_j,D\right)-\mathcal{P}_{mcv}\left(E,D\right)<\varepsilon$. *The theorem is proved* .

Consequence 1. *For any decreasing sequence of compact sets,* $K_1 \supset K_2 \supset \ldots$ *the following right-hand continuity holds:*

$$\mathcal{P}_{mcv}\left(\bigcap_{j=1}^{\infty}K_j,\, D\right)=\lim_{j\to\infty}\mathcal{P}_{mcv}\left(K_j,\, D\right). \tag{5}$$

Consequence 2. *For any increasing sequence of sets* $E_1 \subset E_2 \subset \ldots$, $E=\bigcup_{j=1}^{\infty}E_j$, *there is continuity on the left*

$$\mathcal{P}_{mcv}\left(\bigcup_{j=1}^{\infty}E_j, D\right)=\lim_{j\to\infty}\mathcal{P}_{mcv}\left(E_j, D\right). \tag{6}$$

Theorem 6. *The set function* $\mathcal{P}_{mcv}(E,D)$ *has all the properties of Choquet measurability, and therefore any Borel set is measurable in* $\mathcal{P}_{mcv}$*-capacity . Thus, if* $E \subset D$ *a Borel set,* $E \in B$ *, is measurable in capacity, then its internal and external capacities coincide:* $\mathcal{P}_{*mcv}\left(E,D\right)=\mathcal{P}^{*}_{mcv}\left(E,D\right)=\mathcal{P}_{mcv}\left(E,D\right)$ *, where* $\mathcal{P}_{*mcv}\left(E,D\right)=\sup\left\{\mathcal{P}_{mcv}\left(K,D\right): K \subset E-\text{компакт}\right\}-$ *internal and* $\mathcal{P}^{*}_{mcv}\left(E,D\right)=\inf\left\{\mathcal{P}_{mcv}\left(U,D\right): U \supset E-\text{открытое}\right\}-$ *external capacity of a set* E.

8. CONCLUSION

In conclusion, this paper significantly advances the mathematical framework of the theory of m-convex functions by establishing rigorous metric and capacitive characteristics for geometric domains in D. The structural analysis leads to several key findings:

- **Characterization of** mcv**-Polar Sets:** The work demonstrates that mcv-polar sets are inherently connected to classical potential theory, proving that every mcv-polar set has a Lebesgue and Hausdorff measure of zero. Based on the Trudinger-Wang theorem, it is established that while non-empty mcv-polar sets cannot exist when due to the Holder continuity of the functions, a rich variety of such sets emerges when .
- **Extremal Properties of** $\mathcal{P}_{mcv}$**-Measures:** The paper proves that the introduced $\mathcal{P}_{mcv}$-measure behaves as a real-variable analogue to P-measures in pluripotential theory. It serves as a powerful tool that is identically zero if and only if the underlying set is mcv-polar.

Ultimately, these results offer essential tools for solving Dirichlet problems, evaluating uniform estimates of m-convex functions, and understanding the metric topology of subsets within mcv-convex domains.

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DOI: `10.5281/zenodo.20510189`